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Donis, John Nicholas; Burns, Thomas Morris, Jr.

Monterey, California. Naval Postgraduate School

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JOHN NICHOLAS DONIS
and
THOMAS MORRIS BURNS

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OPTIMUM AIRCRAFT WEAPONS SELECTION AND SEQUENTIAL FIRING STRATEGY FOR TARGETS OF OPPORTUNITY

by

John Nicholas Donis
Lieutenant, United States Navy
A. B., University of Pennsylvania, 1958

and

Thomas Morris Burns
Lieutenant, United States Navy
B. S., Pennsylvania State University, 1959

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[redacted]

ABSTRACT

A pilot engaged in a strike mission against targets of opportunity is continually confronted with the decision as to how many and which of his available weapons should be expended against a variety of targets which are encountered. Associated with this decision problem is the problem of determining an optimal load for such a mission subject to the constraints of available payload and number of weapons stations on the aircraft. Certain assumptions are made concerning the distribution of targets within the target area which lead to a dynamic programming formulation of the decision problem. This yields a system of ordinary differential equations which are solvable recursively. In addition to the dynamic programming model, a sub-optimum determination of a "best load" is discussed. Although not as complete or precise as the dynamic programming method, this formulation is more readily adaptable to squadron level decision making.

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1. INTRODUCTION

The problem of selecting weapons for an aircraft strike mission and using those weapons effectively has been documented quite extensively. However, present publications are devoted almost exclusively to the situation where the target is assumed known prior to the mission and the aircraft can be loaded for that target, or where several different aircraft can be loaded differently to cover an unknown spectrum of targets.[1] These approaches are inadequate for determining weapons requirements against targets of opportunity. A pilot engaged in such a mission is confronted with the decision as to which and how many of his available weapons to expend when any target is encountered. When various types of targets might be encountered, each with a different strategic or tactical value, he wishes to maximize the value of the targets he can expect to destroy. Associated with this is the problem involving the choice of weapons that should be initially loaded onto the aircraft for such a mission. Here the problem is constrained by maximum payload, weapons, and weapons stations availability. It is desired to obtain, within these constraints, (1) the optimum feasible weapons mix to be loaded for use against an uncertain spectrum of targets, and (2) a doctrine for the expenditure of these weapons when targets are encountered.

The model developed herein attacks these problems in reverse order. First, an optimum firing doctrine is generated for all feasible loads. Then that load which yields the maximum expected return,

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[1] NAVWEPS Report 8664, "Preferred Aircraft Loadings Based on the Predicted Combat Effectiveness of Conventional Air-to-Surface Weapons," NOTS TP 3710, 1965, (SECRET).

when expended in accordance with the optimum doctrine, becomes the optimal load.

ASSUMPTIONS IN THE MODELS

Certain assumptions, necessary to the model development are made concerning target distributions within the target area and tactics of the strike aircraft.

It is assumed that there are a finite number of different target types, each randomly and independently distributed within the target area with known density. For example, from photo-reconnaissance or other sources it might be known that the density of trucks within the target area is x per square mile. The number of targets of each type is assumed to be large enough so that the probability of future encounters is unchanged by consideration of the number already met. Further, these targets are assumed to be of a unitary nature, characterized by the existence of single shot kill probabilities, as distinguished from area targets characterized by probabilities of destroying a certain percentage of the area or value of the target.

The assumption is made that a ratio scale of target values can be specified. For example, with the least valuable target as a base with value V, the achievement of a kill on a target type i is worth 7V, target type j is worth 8V, etc.

For defensive reasons the strike aircraft is restricted to a single attack on any target which might be encountered. The pilot may not drop a bomb on the target, assess damage, and then make another attack if a kill has not been achieved. He may however, expend any number of weapons on this single pass provided they are all

the same type. That is, while he may expend three 500 lb. bombs, he may not drop one 500 lb. bomb and one 1,000 lb. bomb. This restriction is not necessary to the development of the models, but rather recognizes that differences in aim points between different weapons make this mixed drop undesirable.

METHOD OF SOLUTION

Using data on weapons weights, constraints of aircraft payload capacity, and the number of available weapons stations on the plane, all feasible loadings can be enumerated. Initial loads are then considered which are not subsets of some other feasible load. That is, if five weapons of type one and three weapons of type two constitute a feasible load, then four weapons of type one and three of type two is also feasible but a subset of the former. The kill potential of some subset of a feasible load is clearly less than that of the original load and is therefore not considered in the determination of an optimal initial loading.

A functional equation for the expected value of targets killed is then set up as a dynamic programming process yielding a value for kill potential for each of the feasible loads being considered. The maximum kill potential from amongst these loadings determines the optimal load for the strike aircraft. Additionally in the solution of this dynamic programming process, the optimal firing strategy for the pilot is obtained for any initial load. The optimized sequential decision process yields a policy which determines as a function of the operating time and weapons remaining, the best weapons to be expended in an encounter with any of the various target types.

Dynamic programming provides a complete and very precise means of determining the optimum load for a "targets of opportunity" type mission. Except in the very simplest cases however, the problem is quite complex and will require a computer for solution. In the majority of operational situations such equipment will not be available. Thus an approximate method, suitable for hand calculation, is needed for determining the initial load.

In order to achieve simplicity, this "hand model" must sacrifice detail and thus completeness. In this respect it is a sub-optimal method. The simplified method is best suited for comparing a few, rather than all, feasible loads for use against a selected target list. By choosing the best weapon for each target type and comparing the expected returns for various mixes of these weapons, a "best load" can be generated.

2. GENERAL DISCUSSION AND DEVELOPMENT OF THE MODELS

The random distribution of targets leads to an exponential inter-encounter time distribution for the strike aircraft searching the target area.[1] (All targets which are within the visibility range of the aircraft's track are assumed sighted. Alternatively the sightings could be reduced by some degradation factor which takes into account the difficulty associated with perceiving a given target type. This however would not alter the analysis which follows.) The mean rate of sightings (the reciprocal of the mean of the exponential distribution of inter-encounter times) is denoted by r.

DYNAMIC PROGRAMMING MODEL

The development of the dynamic programming formulation of the problem is best seen by first considering the simpler cases. To begin with, a model has previously been developed for the allocation of weapons over a limited time interval.[2] This can be utilized for generating the optimal firing strategy in the simplest case of one target type-one weapon type. The optimal loading problem is not pertinent; for a single weapon type the best load is clearly the maximum number that can be carried on the aircraft. First define:

$f_n(t)$ = expected number of targets killed in the time interval $(0,t)$ if there are n weapons at time $t = 0$ and these are utilized optimally.

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[1] Appendix I

[2] Bram, J., "Allocation of Weapons to Targets with Exponential Arrival Times in a Limited Time Interval," OEG Interim Research Memorandum 32, 1963.

An initial loading of N weapons and a time T allowable for search is assumed, and $f_n(t)$ will be calculated for $0 \le n \le N$ and $0 \le t \le T$. Define:

p_j = probability that j weapons expended against a target will achieve a kill.

Clearly $f_n(t)$ is dependent on the p_j's and on r, the encounter rate.

By Bellman's "principle of optimality,"[1] for $0 \le t_1 \le t$, the optimal policy in the interval (0, t), given n weapons at time t = 0, must also be optimal in the subinterval (t_1, t) given $n - n_1$ weapons at time $t = t_1$, where n_1 weapons were expended in the first subinterval $(0, t_1)$.

Let $t_1 = \Delta t \ll T$. By definition of the exponential arrival process, the probability of an encounter in a small interval of time Δt is proportional to Δt with constant of proportionality r, the encounter rate. Thus the probability of one encounter in the time interval $(0, \Delta t)$ is $r\Delta t$ and no encounters occur with probability $1 - r\Delta t$.

Breaking the time interval (0, t) into the two sub-intervals $(0, \Delta t)$ and $(\Delta t, t)$ yields:

$(1 - r\Delta t)$ = the probability that no encounters will occur in the first sub-interval.

p_j = the expected return in the first sub-interval having expended j weapons.

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[1] Bellman, R., Dynamic Programming, (Princeton, New Jersey: Princeton University Press, 1957), p. 83.

$f_n(t - \Delta t)$ = the expected return in the second sub-interval having expended no weapons in the first and following an optimal strategy in $(\Delta t, t)$.

$f_{n-j}(t - \Delta t)$ = expected return in the second time interval having expended j of the n weapons in the first sub-interval and behaving optimally in $(\Delta t, t)$.

The expected return over the whole interval (0, t) is then:

$$(1 - r\Delta t)\ f_n(t - \Delta t) + r\Delta t\ (p_j + f_{n-j}(t - \Delta t))$$

To optimize over the period (0, t), j must be chosen such that this expression is a maximum. Therefore

$$(1) \qquad f_n(t) = (1 - r\Delta t)\ f_n(t - \Delta t) + r\Delta t \max_{1 \le j \le n} (p_j + f_{n-j}(t - \Delta t))$$

The maximization here is over the range $1 \le j \le n$ rather than $0 \le j \le n$ since it clearly would never pay to let a target go by without firing any weapons, saving them all for future encounters which are not certain to occur.

Rearranging (1), dividing by Δt and taking the limit as Δt approaches zero yields:

$$(2) \qquad f_n'(t) = - rf_n(t) + r \max_{1 \le j \le n} (p_j + f_{n-j}(t))$$

From (2), $f_n(t)$ can be calculated for all n and t recursively as follows: By definition, $f_0(t) = 0$ for all t. (The expected return is zero over any period of time if there are no weapons with which to operate.) Also $f_n(0) = 0$ for all n. (The expected return is zero if there is no time in which to operate.) Solving (2) for

$n = 1$ yields:

$$f_1(t) = p_1 (1 - e^{-rt})$$

Note that this result was to be expected. Exponential inter-encounter times lead to a Poisson counting process for the number of encounters within a time interval. The term $(1 - e^{-rt})$, is simply the probability of at least one encounter in the time interval $(0, t)$, and p_1 is the probability that, given an encounter, a kill will be achieved with the expenditure of the single weapon.

Utilizing $f_1(t)$ equation (2) can now be solved for $n = 2$.

$$f_2'(t) = - rf_2(t) + r \max (p_1(2 - e^{-rt}), p_2)$$

Since $p_2 > p_1(2 - e^{-rt})$ when either $t < \frac{1}{r} \ln \frac{p_1}{2p_1 - p_2}$ or if $p_2 > 2p_1$, the solution to the differential equation is:

$$f_2(t) = p_2 (1 - e^{-rt}) \quad \text{for } t \le \frac{1}{r} \ln \frac{p_1}{2p_1 - p_2} \quad \text{or if } p_2 \ge 2p_1$$

$$f_2(t) = 2p_1 - e^{-rt} \left(rp_1 t + p_1 + p_2 - p_1 \ln \frac{p_1}{2p_1 - p_2}\right)$$

for $t \ge \frac{1}{r} \ln \frac{p_1}{2p_1 - p_2}$ and $p_2 < 2p_1$

The significance in this solution lies in the "cross-over" point in time. If the search time remaining is less than this value of t, the optimal return is achieved by dropping both bombs on the next target which is encountered. If greater, only one bomb is dropped on the target, the other being saved for possible future encounters.

Having found $f_2(t)$, $f_3(t)$ can now be calculated. The differential equation however becomes unduly complicated for $n > 2$. A numerical solution may be obtained for the problem though by recourse to equation (1), the difference equation, with which $f_n(t)$ can be calculated for any n to any desired degree of accuracy by taking Δt small enough.

With the possibility of encounters with targets of different types, the value of a target which is met must be considered when the decision is made as to how many weapons should be expended.

The methods used in developing the functional equation are analagous to those for the simpler model with the addition of considerations of target value and different kill probabilities. Define:

m = the number of different target types.

p_{ij} = the probability that j weapons expended on a target type i will achieve a kill.

r_i = average encounter rate with type i targets.

v_i = value of target type i.

$f_n(t)$ = expected value of targets killed in the interval (0, t) given n bombs at time t = 0 and optimal utilization of these weapons.

Then:

$$(3) \quad f_n(t) = \left(1 - \sum_{i=1}^{m} r_i \, \Delta t\right) f_n(t - \Delta t) + \sum_{i=1}^{m} r_i \, \Delta t \max_{0 \le j \le n} \left(v_i p_{ij} + f_{n-j}(t-\Delta t)\right)$$

Note that in this case the maximization is over the range $0 \le j \le n$ vice $1 \le j \le n$. Here the expected return might be greater if no weapons are dropped on targets of little value, expecting at some later time in the mission to meet something of greater worth.

The differential equation is obtained by subtraction of $f_n(t)$ from both sides of the above equation, division by Δt and taking the limit as Δt approaches zero.

$$f'_n(t) = -\sum_{i=1}^{m} r_i f_n(t) + \sum_{i=1}^{m} r_i \max_{0 \le j \le n} (v_i p_{ij} + f_{n-j}(t))$$

For n = 1 and m = 2 the solution to this equation can be shown to be:

$$f_1(t) = \frac{(r_1 p_1 v_1 + r_2 p_2 v_2)}{(r_1 + r_2)} (1 - e^{-(r_1 + r_2)t}) \text{ for } p_2 v_2 > p_1 v_1$$

and

$$0 \le t \le \frac{1}{r_1 + r_2} \ln \frac{(r_1 p_1 v_1 + r_2 p_2 v_2)}{r_2 (p_2 v_2 - p_1 v_1)}$$

or if $p_1 v_1 > p_2 v_2$ then for

$$0 \le t \le \frac{1}{r_1 + r_2} \ln \frac{(r_1 p_1 v_1 + r_2 p_2 v_2)}{r_1 (p_1 v_1 - p_2 v_2)}$$

or if $p_1 v_1 = p_2 v_2$ then for all t.

$$f_1(t) = p_1 v_1 + (p_2 v_2 - p_1 v_1) \left[\frac{r_1 p_1 v_1 + r_2 p_2 v_2}{r_1 (p_1 v_1 - p_2 v_2)} \right]^{\frac{r_1}{r_1 + r_2}} e^{-r_1 t}$$

for $p_1 v_1 > p_2 v_2$ and $t \ge \frac{1}{r_1 + r_2} \ln \frac{r_1 p_1 v_1 + r_2 p_2 v_2}{r_1 (p_1 v_1 - p_2 v_2)}$

$$f_1(t) = p_2 v_2 + (p_1 v_1 - p_2 v_2) \left[\frac{r_1 p_1 v_1 + r_2 p_2 v_2}{r_2 (p_2 v_2 - p_1 v_1)} \right]^{\frac{r_1}{r_1 + r_2}} e^{-r_2 t}$$

for $p_2 v_2 > p_1 v_1$ and $t \ge \frac{1}{r_1 + r_2} \ln \frac{r_1 p_1 v_1 + r_2 p_2 v_2}{r_2 (p_2 v_2 - p_1 v_1)}$

Thus for the simple case of two target types and a single weapon, the expected returns can be calculated analytically. In addition, the optimal strategy is found, here consisting merely of a cross-over time, the time before which it does not pay to fire the weapon at the target with the lowest $p_i v_i$. For $n > 1$ however, it can readily be seen that solutions to the differential equation are not so easily obtained. Again though, through recourse to the difference equation, (3), numerical solutions can be obtained for any n to any desired degree of accuracy.

Further generalization of the model entails a relaxation of the restriction to a single weapon type. The functional equation is similar to those for the simpler models with some minor changes in the definitions of the terms. Define:

p_{ijk} = the probability that k weapons of type j expended on a target of type i will achieve a kill.

L = the number of different weapon types.

$f_{n_1 n_2 n_3 \cdots n_L}(t)$ = expected value of targets killed in the interval (0, t), given n_i weapons of type i (for $i=1,\cdots,L$) at $t = 0$ and optimal utilization of these weapons.

The difference equation can be written:

$$f_{n_1 n_2 n_3 \ldots n_L}(t) = (1 - \sum_{i=1}^{m} r_i \Delta t)\, f_{n_1 n_2 n_3 \ldots n_L}(t - \Delta t)$$

$$+ \sum_{i=1}^{m} r_i \Delta t \max_{1 \le j \le L} \max_{0 \le k \le n_j} (v_i p_{ijk} + f_{n_1 n_2 \ldots n_j - k \ldots n_L}(t - \Delta t))$$

The differential equation can be written as before, but the solutions are obtainable only in the most trivial cases.

A FORTRAN program has been written however from which numerical solutions to the difference equation can be calculated.[1] With inputs of the aircraft load carrying capabilities, the mission time, and the parameters of the model (m, r_i, v_i, L, p_{ijk}, and Δt) the program selects values for the n_i which are consistent with the limitations of the aircraft and maximize f. Additionally, another FORTRAN program[1] lists the optimal strategy to be employed in the decisions relating to the expenditure of these weapons.

A SUB-OPTIMAL DETERMINATION OF LOAD

In the majority of practical cases hand calculation will be the only means available for determining loading requirements. In such cases a sub-optimal method can be used at the squadron level for obtaining the initial weapons load for a mission. Define:

T = search time (i.e., mission time allotted to searching for and attacking targets).

$p_{ni}(T)$ = probability that target type i will be met n times during the mission.

$\Pi_i(T) = \sum_{n=1}^{\infty} p_{ni}(T)$ = probability that target type i will be met at least once during the mission.

v_i = value of target type i.

m = number of target types.

b_{js} = number of sticks[1] of s weapons of type j in the load.

L = number of weapon types available.

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[1]In this development the terms stick and salvo are used equivalently. The choice of a stick or a salvo, in the load determination, will depend upon which gives the higher kill probability.

k_{iws} = probability that a stick of s type w weapons will kill target type i.

Note that in this development individual bombs are considered as a stick of one; i.e., two 250 lb. bombs might be considered as two sticks of one type j or one stick of two type j weapons.

If, a priori, only the more highly valued targets and those individual weapons with the highest kill probabilities against these targets are considered,[1] the assumption can be made that a target sighted will be dropped on, as long as the appropriate weapon for that target is available. (Note that this assumption is used only for determining the load. In the actual performance of the mission there is the possibility that, if that weapon most effective against a target is not available, a less effective weapon will be dropped.) The problem then becomes one of maximizing, for a given mission time, the expected value of targets killed, E, for those loads being considered. E may be determined as follows:

$$E = \sum_{i=1}^{m} f_{ijs} \, k_{ijs} \, v_i$$

where f_{ijs} is the expected number of type i targets on which weapon type j can be expended, and j indicates that weapon type for which

$k_{ij1} \geq k_{iw1}$ for all w = 1, 2, $\cdots$, L.

If the number of type j weapons available were limitless, f_{ijs} would simply be equal to $\bar{n} = \sum_{n=o}^{\infty} np_{ni}(T)$. However, since a given load is limited to b_{js} sticks of type j weapon, no more than b_{js} can be dropped regardless of how many type i targets are sighted. Thus

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[1]The decision as to which targets and weapons will be considered will depend highly upon the judgement of the individual responsible for determining the aircraft load.

f_{ijs} is given by

$$(4) \qquad f_{ijs} = \sum_{n=o}^{b_{js}-1} np_{ni}(T) + \sum_{n=b_{js}}^{\infty} b_{js}p_{ni}(T)$$

Equation (1) can be reduced to a more easily calculable form.

Since $\sum_{n=o}^{b_{js}-1} np_{ni}(T) = \sum_{n=1}^{b_{js}-1} np_{ni}(T)$ and $\sum_{n=b_{js}}^{\infty} p_{ni}(T) = \Pi_i - \sum_{n=1}^{b_{js}-1} p_{ni}(T)$,

equation (4) can be written as

$$f_{ijs} = \sum_{n=1}^{b_{js}-1} np_{ni}(T) + b_{js}\Pi_i - b_{js} \sum_{n=1}^{b_{js}-1} p_{ni}(T)$$

$$= b_{js}\Pi_i - \sum_{n=1}^{b_{js}-1} (b_{js}-n)\, p_{ni}(T)$$

but $\Pi_i = 1 - p_{0i}(T)$ and consequently $b_{js}\Pi_i = b_{js} - b_{js}p_{0i}(T)$. Note that $b_{js}p_{0i}$ can be expressed as $(b_{js} - 0)\, p_{0i}$. Thus equation (4) now reduces to

$$(5) \qquad f_{ijs} = b_{js} - \sum_{n=o}^{b_{js}-1} (b_{js}-n)\, p_{ni}(T)$$

Equations (4) and (5) are valid for all $b_{js} \geq 1$. By definition $f_{ijs} = 0$ for $b_{js} = 0$, since no sticks of type j weapon can be dropped if none are carried.

As noted before, the random distribution of targets and consequent exponential inter-encounter times lead to a Poisson counting

process in determining the probabilities of encountering given numbers of targets; i.e., for type i targets appearing at a mean rate r_i,

$$p_{ni}(T) = e^{-r_i T} \frac{(r_i T)^n}{n!} .$$

This gives

$$(6) \qquad f_{ijs} = b_{js} - \sum_{n=0}^{b_{js}-1} (b_{js}-n)\, e^{-r_i T} \frac{(r_i T)^n}{n!}$$

Table 11, Appendix III gives values of f_{ijs} for various combinations of b_{js} and $r_i T$. This table greatly facilitates calculation of a "best load" from the feasible loads being considered.

To determine the "best load" for a given set of targets, find that individual weapon type j, for each target, which has the highest kill probability. The loads considered are then the different mixes of these weapons selected from a list of feasible loads for the aircraft.

For a given load, b_{js} will be known for each type weapon in the load and the encounter rate r_i will be estimated for each of the critical targets. The planned search time T will also be known for the mission. Thus using known values of k_{ijs} and estimated values of v_i, E can be calculated.

Consider first the case where weapons will be dropped individually (sticks of one only) and where each weapon type in the load will be used against only one type target. In other words, target types and weapon types are matched one to one. Since weapons are to be dropped singly, $b_{js} = b_{j1}$ for each weapon type j. To determine E for such a load, first determine b_{j1} and $r_i T$ for each weapon-target type combination. Then obtain f_{ijs} from Table 11, and multiply this

value by $k_{ijs} v_i$. Let $f_{ijs} k_{ijs} v_i = E_{ij}$. Add the resulting E_{ij}'s to get E for the load, i.e., $E = \sum_{i=1}^{m} E_{ij}$. (Note that the E_{ij}'s are summed over i only, since the subscript j simply indicates that weapon which is most effective against target type i.) In the same manner, E may be determined for each load being considered; that load which yields the maximum E is then the "best load."

For this sub-optimal model, the extension of the problem to include sticks of two or more weapons is accomplished quite easily, since different size salvos or sticks of a given type weapon constitute different loads. For example, as noted previously, two 250 lb. bombs might be considered as a load consisting of two sticks of one bomb or as another load consisting of one stick of two bombs. With this in mind, E can be calculated for these loads in the same manner as previously outlined. In order to keep the problem hand workable, this approach assumes that each delivery of a given type weapon will consist of the same size stick. This assumption is made solely for determination of the aircraft loading prior to flying the mission. During the actual mission, however, it is quite likely that sticks of different sizes will be expended at different times. For example, a pilot might drop one type j bomb on the first truck he sees, a stick of three type j bombs on the second truck he sees, etc. Prior to the mission, however, this load of four type i weapons could have been evaluated as one stick of four, two sticks of two, or possibly as four sticks of one, since calculations for sticks of varying sizes quickly exceed the bounds of practical hand workability.

In addition to the case where targets and weapons are matched one to one, consideration must be given to the case where a given weapon type is optimal for more than one target type. If the assumption that a target sighted will be dropped on were adhered to, the expected number of type i targets killed would be proportional to the encounter rate r_i. However, in this situation, a concession is made to the fact that, early in the mission, a target of lesser value might be passed in hopes of encountering a more highly valued target later. Thus it is assumed that the expected number of type i targets killed is proportional to both the encounter rate, r_i, and the target value, v_i. The expected return from a given load is now calculated exactly as before with the following minor modification: For all targets type i against which weapon type j is most effective, calculate the E_{ij}'s exactly as in the simpler case (as if all type j weapons were to be expended on each type target). Then to approximate the particular expected value of type y targets killed by weapon type j, E_{yj} is multiplied by the normalizing factor $N_y = \frac{r_y v_y}{\Sigma r_i v_i}$ where the summation in the denominator is taken over those type i targets for which weapon type j is optimal. These expected values are then added to give E as before.

A logical extension to this hand approximation is the consideration of "secondary" weapons. A secondary weapon would be considered to be a weapon which has high kill probabilities against several targets, yet is optimal against none of them. There are certainly cases where such a weapon might give higher expected returns over the entire target spectrum than would a load consisting of only those weapons which are optimal for the individual targets. For example,

consider a case in which there are two targets and three weapons to be used against these targets. The single stick kill probabilities for the individual weapons are as follows:

WEAPON	TARGET 1	TARGET 2
1	.09	.01
2	.01	.09
3	.08	.08

If the load were limited to those weapons which are optimal for individual targets, only weapons 1 and 2 would be considered. However, it is obvious that weapon 3 merits examination, and, in fact, it appears as if a pure load of weapon type 3 might possibly be the best load to use in this particular case. Calculation of E for such a secondary weapon is identical to that in the foregoing case in which one weapon is considered for use against more than one target.

This sub-optimal development may, at first glance, appear to be long and cumbersome. It must be realized though that in most actual situations all but a very few feasible loads will be eliminated from consideration prior to any calculation, and these will be examined only as far as they pertain to a small number of critical target types (normally not more than three or four). Thus it can be seen that relatively few calculations will actually be required and these will be of a trivial nature.

In order to clarify the use of the sub-optimal model, the following simple example is presented in detail.

Consider an armed reconnaissance mission with a planned search time of one hour. (Thus $T = 1$ so that $r_i T = r_i$ for all targets).

Three target types are considered:

$$v_1 = 2 \qquad r_1 = 5$$

$$v_2 = 1 \qquad r_2 = 3$$

$$v_3 = 4 \qquad r_3 = 2$$

The aircraft has two weapon stations and two types of weapons are considered, one more effective against target type 1, the other more effective against types 2 and 3 (see Table 2).

The possible loads are indicated in Table 1.

	WEAPON			
	1		2	
Load	Single	Stick of 2	Single	Stick of 2
A	1		1	
B	2			
C			2	
D		1		
E				1

TABLE 1
COMPOSITION OF POSSIBLE LOADS

j	s \ i	1	2	3
1	1	.02	.03	.01
	2	.06	.08	.03
2	1	.01	.05	.02
	2	.03	.12	.05

TABLE 2
KILL PROBABILITIES

Load A: $b_{11} = 1$, $b_{21} = 1$

Target 1 - Weapon 1

$f_{111} = .99326$ (from Table 11; $b_{js} = 1$, $r_i T = 5$)

$E_{11} = f_{111} k_{111} v_1 = (.99326)\ (.02)\ (2) = .0397$

Target 2 - Weapon 2

$f_{221} = .95021$ ($b_{js} = 1$, $r_i T = 3$)

$E_{22} = f_{221} k_{221} v_2 = (.95021)\ (.05)\ (1) = .0475$

Target 3 - Weapon 2

$f_{321} = .86466$ ($b_{js} = 1$, $r_i T = 2$

$E_{32} = f_{321} k_{321} v_3 = (.86466)\ (.02)\ (4) = .0692$

Since weapon type 2 is optimal for two target types, E_{22} and E_{32} must be normalized.

$N_2 E_{22} = (r_2 v_2 / r_2 v_2 + r_3 v_3)\ E_{22} = (3/11)\ (.0475) = .0129$

$N_3 E_{32} = (8/11)\ (.0692) = .0608$

Thus for load A, $E = .0397 + .0129 + .0608 = \underline{.1134}$

Load B: $b_{11} = 2$

$f_{111} = 1.95283$ ($b_{js} = 2$, $r_i T = 5$)

$E_{11} = E$ (Load B) $= (1.95283)\ (.02)\ (2) = \underline{.0724}$

Load C: $b_{21} = 2$

$f_{221} = 1.75106$ ($b_{js} = 2$, $r_i T = 3$)

$N_2E_{21} = (3/11)\ (1.75106)\ (.05)\ (1) = .0240$

$f_{321} = 1.45866 \quad (b_{js} = 2,\ r_iT = 2)$

$N_3E_{32} = (8/11)\ (1.45866)\ (.02)\ (4) = .0846$

$E(\text{Load C}) = .0240 + .0846 = \underline{.1086}$

Load D: $b_{12} = 1$ (Note that this is almost the same as Load B, but now the two weapons are considered as one stick of two vice two sticks of one).

$f_{112} = .99326$

$E_{11} = E(\text{Load D}) = (.99326)\ (.06)\ (2) = \underline{.1192}$

Load E: $b_{22} = 1$

$f_{222} = .95021$

$N_2E_{22} = (3/11)\ (.95021)\ (.12)\ (1) = .0312$

$f_{322} = .86466$

$N_3E_{22} = (8/11)\ (.86466)\ (.05)\ (4) = .1252$

$E\ (\text{Load E}) = .0312 + .1252 = \underline{.1564}$

For the loads considered, the maximum E is .1564. Therefore the "best load" is load E which consists of one stick of two type 2 weapons. Note that if sticks of one only (case 1 in the model development) had been considered, the "best load" would have been load A with one of each type weapon yielding E = .1134.

The foregoing sub-optimal method is limited in that it considers search time as a whole rather than a time-stepped firing doctrine as

in the dynamic programming method. Thus those added values which might be generated by strategies which vary with mission time are lost in this formulation. Consequently the expected kill values will generally be much lower than those of the dynamic program formulation, and the "best load" generated will not always be the optimum load. However, the load obtained by this method should, if expended in accordance with the optimum firing doctrine of the dynamic program, be one of the more highly valued loads.

3. SAMPLE RESULTS

OPTIMAL LOAD AND FIRING STRATEGY DETERMINATION

For a mission time of 15 minutes, seven weapons from which to choose, and values of the other input parameters as listed in Table 3, an optimal loading has been determined using the "BESTLD" program of Appendix II, with the following inputs:

Weapon Type :	1	2	3	4	5	6	7
Weapon Weight:	265	300	510	600	675	950	1975

Number of racks = 5

Max payload = 5,000 lbs.

Number of target types = 4

Mission time = 0.25 hours

Delta T = .010 hours (This is the Δt of equation (3)).

TARGET TYPE	ENCOUNTER RATE	VALUE
1	30.00000	1.00
2	1.00000	25.00
3	.10000	60.00
4	3.00000	5.00

KILL PROBABILITIES

		NUMBER OF WEAPONS =	1	2	3	4	5
TARGET	1	WEAPON 1	.020	.040	.060	.070	.080
TARGET	1	WEAPON 2	.030	.040	.060	.070	.080
TARGET	1	WEAPON 3	.040	.070	.090	.100	.110
TARGET	1	WEAPON 4	.005	.008	.010	.011	.012
TARGET	1	WEAPON 5	.030	.050	.070	.071	.080
TARGET	1	WEAPON 6	.050	.050	.100	.120	.130
TARGET	1	WEAPON 7	.120	.160	.180	.200	.202
TARGET	2	WEAPON 1	.010	.020	.024	.030	.035
TARGET	2	WEAPON 2	.020	.030	.040	.050	.060
TARGET	2	WEAPON 3	.012	.021	.030	.036	.042
TARGET	2	WEAPON 4	.016	.032	.047	.061	.076
TARGET	2	WEAPON 5	.022	.039	.053	.065	.075
TARGET	2	WEAPON 6	.007	.012	.016	.019	.021
TARGET	2	WEAPON 7	.050	.150	.153	.200	.240
TARGET	3	WEAPON 1	.050	.080	.140	.170	.190
TARGET	3	WEAPON 2	.094	.143	.172	.196	.205
TARGET	3	WEAPON 3	.060	.090	.150	.190	.220
TARGET	3	WEAPON 4	.036	.070	.102	.131	.161
TARGET	3	WEAPON 5	.119	.175	.220	.233	.245
TARGET	3	WEAPON 6	.070	.100	.160	.200	.230
TARGET	3	WEAPON 7	.080	.120	.180	.220	.250
TARGET	4	WEAPON 1	.011	.020	.028	.035	.042
TARGET	4	WEAPON 2	.019	.036	.048	.056	.064
TARGET	4	WEAPON 3	.019	.032	.042	.051	.057
TARGET	4	WEAPON 4	.001	.001	.001	.001	.002
TARGET	4	WEAPON 5	.032	.054	.064	.082	.090
TARGET	4	WEAPON 6	.009	.016	.023	.028	.033
TARGET	4	WEAPON 7	.029	.057	.066	.080	.090

TABLE 3
KILL PROBABILITIES

These weapons' weights and aircraft payload and rack capacities result in approximately 400 possible loadings which must be considered. From amongst these 400, Table 4 lists the five loads with the greatest expected returns and for comparison the loading yielding the least. A loading designated 1200002 denotes one weapon of type one, two weapons of type two, zero weapons of types three through six, and two weapons of type seven.

LOAD NO.	LOADING	EXPECTED RETURN
1	0300002	1.29514
2	1200002	1.25048
3	0100102	1.22353
4	1000102	1.20670
5	3000002	1.20409
381	2011010	.38069

TABLE 4
LOADINGS AND EXPECTED RETURNS,
DYNAMIC PROGRAMMING METHOD

The expected returns from the optimal load are over three times higher than those of the worst. In fact the twentieth best load, 0020300, had expected returns of only .77700. Thus the optimal loading yields returns more than 65% better than a loading which is better than 95% of the possible loads.

The above inputs and constraints have also been used in the sub-optimal model to generate a "best load." Weapon 7 is the best single weapon against targets 1 and 2, and weapon 5 is best against targets 3 and 4. Thus the only loads considered in this formulation were 0000500, 0000401, and 0000102. Table 5 lists six possible load considerations (including stick options) and the expected returns from each.

Loading	E
0000102 (Weapon 7 used as a stick of 2)	.5766
0000401 (Weapon 5 used as a stick of 4)	.4458
0000102 (Weapon 7 used as 2 sticks of 1)	.3824
0000500 (Weapon 5 used as a stick of 5)	.3740
0000401 (Weapon 5 used as 4 sticks of 1)	.3320
0000500 (Weapon 5 used as 5 sticks of 1)	.1432

TABLE 5
POSSIBLE LOADS AND EXPECTED
RETURNS, SUB-OPTIMAL METHOD

The "best load" using the sub-optimal hand method is 0000102 considering weapon 7 as one stick of two. Although only the principal weapon for each target has been considered, note that weight constraint still allows the loading of one more of the lighter, less effective weapons (weapon 2 rather than weapon 1 since weapon 2 is dominant over the entire target spectrum). The load thus generated corresponds to the third most valuable dynamic programming load, and, if used in accordance with the dynamic programming firing policy, it would be expected to yield a return of greater than 90% of that given by the actual optimum load.

With the best loading as generated by either method, the optimal strategy can be listed using the "POLICY" program of Appendix II. The results of this determination for loading 0300002 are shown in Figure 1 along with a format for displaying the strategy compactly. The time axis is mission time remaining and entries such 2x7 denote that two weapons of type seven should be expended.

As an example, as the mission begins, the subloading is 0300002. If the first target which is met is a type one, and is encountered with

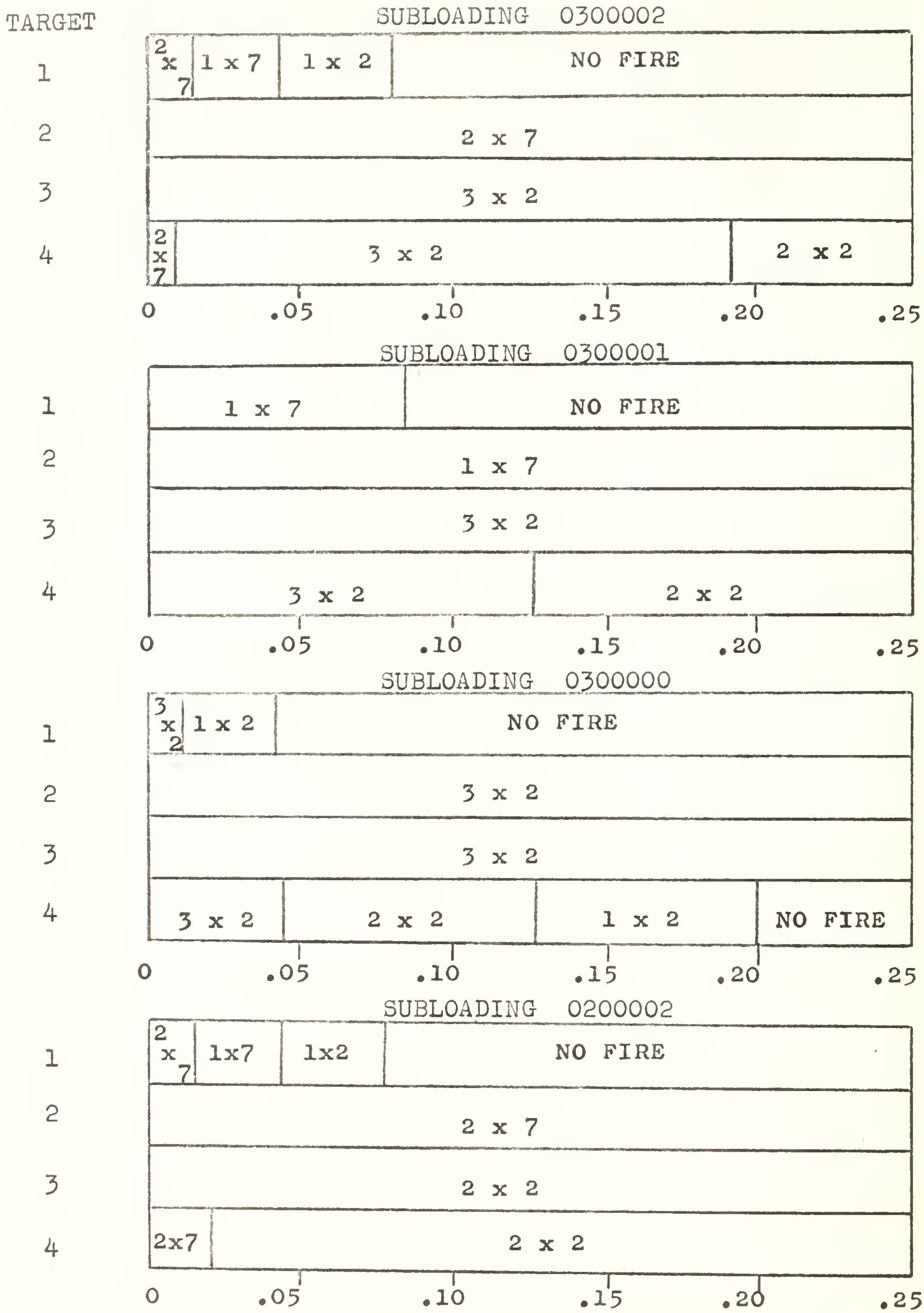
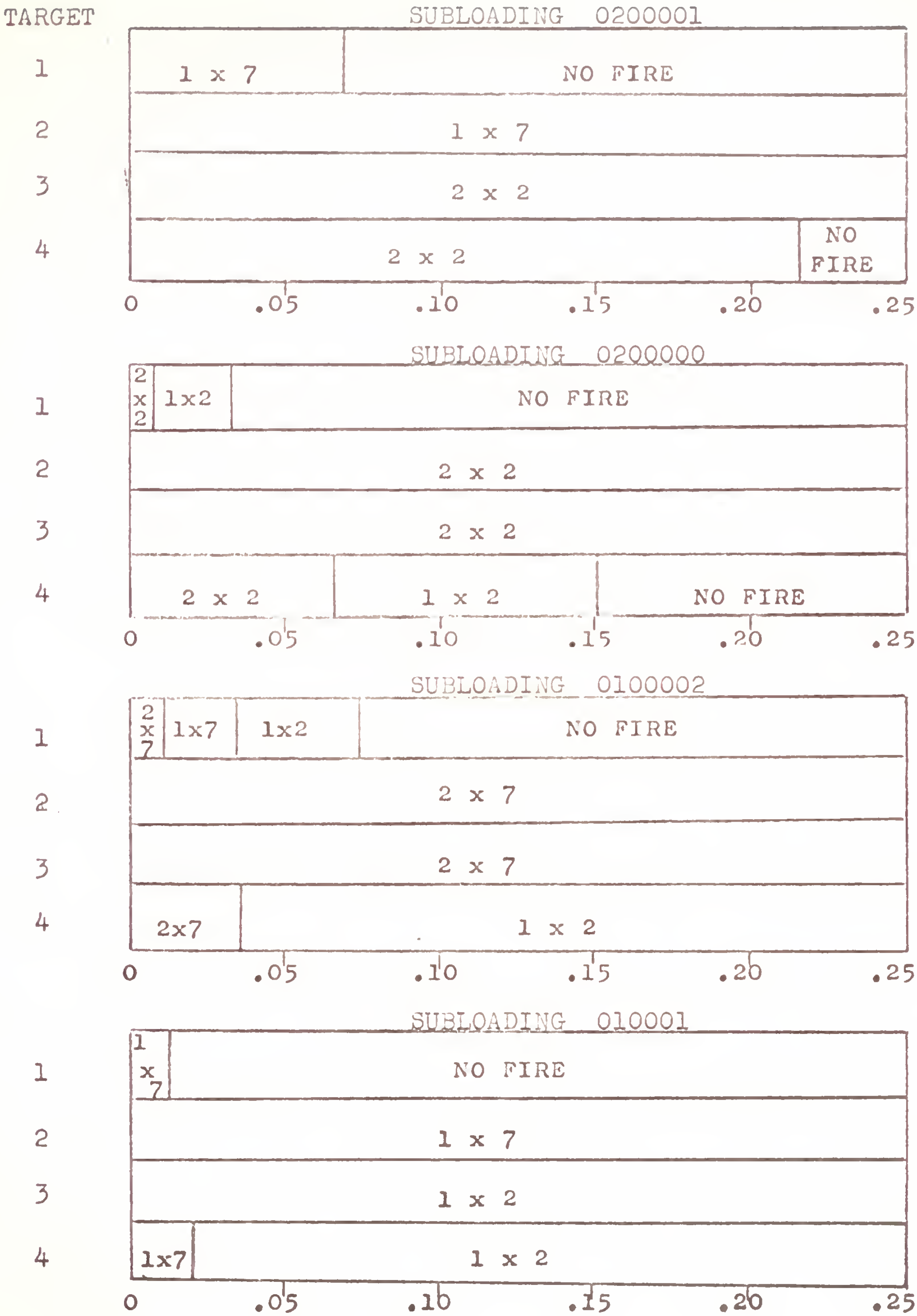
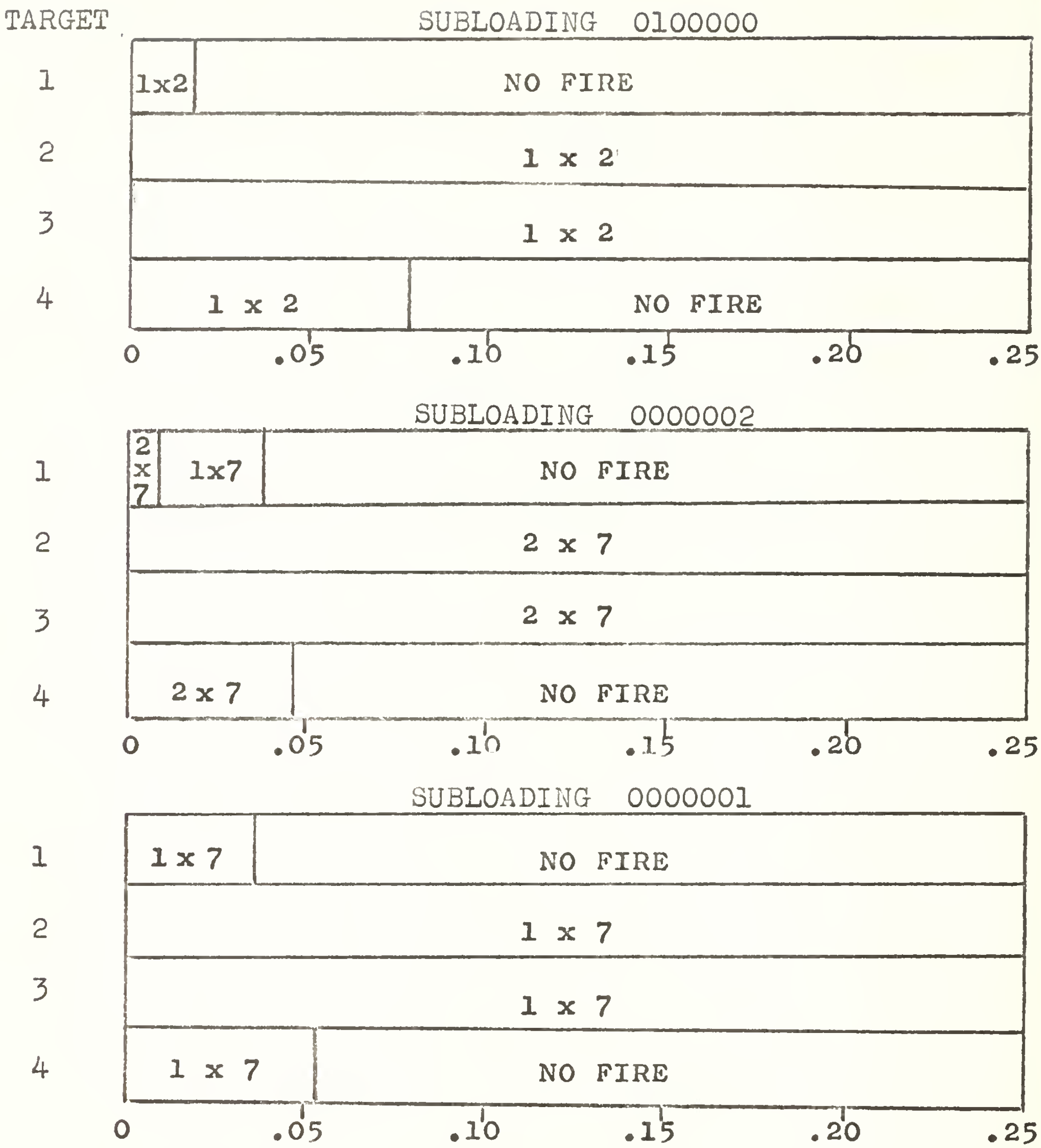


FIGURE 1
OPTIMAL STRATEGY DISPLAY


TARGET
SUBLOADING 0200001
1
2
3
4
1 x 7
NO FIRE
1 x 7
2 x 2
2 x 2
NO FIRE
0
.05
.10
.15
.20
.25
SUBLOADING 0200000
2x2
1x2
NO FIRE
2 x 2
2 x 2
2 x 2
1 x 2
NO FIRE
SUBLOADING 0100002
2x7
1x7
1x2
NO FIRE
2 x 7
2 x 7
2x7
1 x 2
SUBLOADING 010001
1x7
NO FIRE
1 x 7
1 x 2
1x7
1 x 2



(The abscissa in these figures is the time remaining in which to search for and attack targets.)

.22 hours remaining, the optimal strategy indicates that no weapons should be fired. If the next target which is encountered is a type four at time .17, the subloading still being 0300002, a drop of three weapons of type two is called for. The sub-loading now is 0000002. If a type three target is now met, both of the remaining type seven weapons should be dropped.

It is of interest to note the changes which occur over time in the optimal weapon to drop when a target is met. With the loading 0300002 and target type one, with more than .08 hours remaining, no weapons should be expended. With .045 to .08 hours remaining, the optimal strategy calls for the expenditure of one weapon of type two. Weapon type seven however has a higher kill probability against type one targets! The type seven weapons at this point are being saved for possible future encounters with targets of type two and four, both of which are more valuable and occur frequently enough so that there is a significant probability of an encounter in the time remaining. Although weapon type two is the most potent against type three targets -the most valuable target type- the probability of meeting a type three target is rather small because of the low encounter rate of 0.10. It is for this reason that type two weapons are not being conserved. Later in the mission however, the optimal weapons to drop on a type one target are type sevens. At this point in time the probability of future encounters with more valuable targets is remote.

Since the ultimate objective is the maximization of the expected returns for the mission, a consideration might be made for lengthening the search time at the expense of lower payload and weapon station availability. That is, perhaps an additional tank of fuel can be

carried on one of the aircraft's weapon racks, thus reducing the number available on which ordnance can be carried and also reducing the weapons payload, but increasing the search time.

Table 6 lists the expected returns for the five best loads generated when the search time has been increased to 0.50 hours, the payload decreased to 4000 lbs., the number of weapon racks decreased to four, and the other input parameters unchanged from those of Table 3.

LOADING	EXPECTED RETURNS
0000002	1.78803
0000301	1.42184
0100201	1.30025
0010201	1.29748
0000400	1.28808

TABLE 6
BEST LOADS AND EXPECTED RETURNS

Since the expected value of targets killed for the longer mission is significantly better than that for the shorter, 1.78803 compared to 1.29514, it is clearly preferrable to make the trade-off of gas for weapons.

An interesting feature of the kill probability matrix of Table 3 is the preferred weapons with which to attack the most valuable targets. Weapon type seven is the most potent against type two targets while weapon type five is preferred against type three targets. Target type three, although the most valuable, has a very low encounter rate. This results in the fact that weapon type five does not appear frequently in the loadings which yield high expected returns for short missions. Letting the maximum payload equal 4000 lbs. and the number of weapons stations equal four, the preferred loadings of these two

weapons alone can be examined as a function of search time. The three loadings which must be considered are 02, 31, and 40, where now, 02 denotes a load consisting of zero weapons of type five and two weapons of type seven. Calculations show that an initial loading of 02 is optimal for mission times of less than 1.6 hours. For mission times of 1.6 to 8.9 hours, loading 31 is optimal, and finally for $T > 8.9$ hours, loading 40 is preferred.

COMPARISON OF EXPECTED RETURNS FROM NON-OPTIMAL FIRING STRATEGIES

The gains in kill potential realizable through the utilization of the optimal firing doctrine can be demonstrated by comparisons with the expected returns from other strategies. As an example, consider the following case with non-optimal strategies:

Let:

Number of weapons = 1

Operating time = 1 hour

No. of target types = 2

$v_1 = 3$

$v_2 = 4$

Strategy A: Expend the weapon on the first target which is encountered.

E_A = the expected returns from strategy A.

Strategy B: Expend weapon only if a target of type 2 is encountered. (Note that target type 2 is the more valuable).

E_B = the expected returns from strategy B.

T_i: The random variable describing the time to first encounter with target type i. (T_i is exponentially distributed with mean $1/r_i$.)

$$E_A = p_1 v_1 \text{Prob. } (T_1 < \min (t_2, 1 \text{ hour})) + p_2 v_2 \text{Prob. } (T_2 < \min (t_1, 1 \text{ hour}))$$

$$E_A = p_1 v_1 \int_0^1 r_1 e^{-r_1 t_1} \int_{t_1}^{\infty} r_2 e^{-r_2 t_2} dt_2 dt_1 + p_2 v_2 \int_0^1 r_2 e^{-r_2 t_2} \int_{t_2}^{\infty} r_1 e^{-r_1 t_1} dt_1 dt_2$$

$$E_B = p_2 v_2 \text{Prob. } (T_2 < 1 \text{ hour})$$

$$E_B = p_2 v_2 \int_0^1 r_2 e^{-r_2 t_2} dt_2$$

For several values of the parameters p_i and r_i, Table 7 shows the results of calculations of the expected returns using either of these strategies or the optimal.

	Expected Value of Targets Killed: Strat. A	Strat. B	Optimal Strat.	% Gain Optimal Over: A	B
$p_1 = .3$ $p_2 = .4$ $r_1 = 10$ $r_2 = 1$	.963	1.011	1.270	32%	26%
$p_1 = .3$ $p_2 = .4$ $r_1 = 10$ $r_2 = .25$	.916	.354	1.018	11%	185%
$p_1 = .1$ $p_2 = .4$ $r_1 = 10$ $r_2 = 1$	.414	1.011	1.064	150%	5%

TABLE 7
GAINS IN EXPECTED VALUE OF TARGETS KILLED ACHIEVED BY UTILIZING THE OPTIMAL FIRING STRATEGY

These results are representative of those obtainable using other values for the input parameters. Generally, when one of these non-optimal strategies approaches the optimal in expected returns, the other yields very poor results. When the non-optimal strategies yield roughly equivalent returns, the optimal dominates both by a significant amount; this is demonstrated in the first example of the table by gains of 26% and 32% through use of the optimal strategy.

Comparisons can be made in this way with any conceivable strategy. It seems reasonable too that techniques which consider the target values and encounter rates could yield returns comparable to those achievable through use of the optimal strategy. The determination of these strategies however would probably involve calculations no less complex than those of the dynamic program.

SENSITIVITY OF EXPECTED RETURNS TO TARGET VALUES

A complete parametric analysis is beyond the scope of this paper; however, some further comment must be made about target values. The developments of both the dynamic programming model and the hand approximation are based upon the assumption that a ratio scale of values can be assigned to all possible targets. Such a determination is completely subjective and must rely solely on the judgement of experienced planners. Quantitative target values might be determined by presenting a list of targets to several military commanders and asking them to rate these targets according to military value. Targets could be assigned numbers consistent with their importance relative to a target of least value. The results of such a large number of opinions could then be analyzed by methods of sensory psychophysics in order to assign representative

values to all targets. Methods for accomplishing such an analysis have been reviewed and consolidated by S. S. Stevens.[1]

The above method will obviously generate strategic values. These will certainly not be valid for every tactical situation (e.g., a tank about to overrun friendly troops is more important than a railroad bridge at that moment even though the bridge may have a higher strategic value). A strategic scale will, however, provide a starting point -- a base which can be adjusted to meet specific tactical situations.

An analysis of the effects on the expected returns of incorrect selections of target values can be made rather easily for the simple case of a single weapon and two target types. Assume the "true" ratio of the target values is V_r^*. Now suppose that an erroneous ratio, V_r, has been used to calculate the strategy. The expected returns which result from the utilization of this strategy - these returns being computed using the true value ratio, V_r^* - can be compared with the maximum of the expected returns, this maximum being obtained when the strategy derived using V_r^* is utilized. Define:

$$V_r = v_2/v_1$$

and let the mission time = 1 hour. With a single weapon, a strategy consists merely of a cross-over time, the time after which the least valuable target should be attacked if met. These times can be calculated from:

$$t = \frac{1}{r_1+r_2} \ln \frac{r_1p_1 + r_2p_2V_r}{r_2(p_2V_r - p_1)} \quad \text{if } V_r > 1$$

$$t = \frac{1}{r_1+r_2} \ln \frac{r_1p_1 + r_2p_2V_r}{r_1(p_1 - p_2V_r)} \quad \text{if } V_r < 1$$

$$t = \infty \quad \text{if } V_r = 1$$

---

[1]S.S. Stevens, "A Metric for Social Concensus," Science, vol. 151, 4 February 1966, pp. 530-541.

Using these cross-over times then, the expected returns, E, are calculated from:

$$E = p_2 V_r^* \frac{r_2}{r_1+r_2}\left(1 - e^{-(r_1+r_2)}\right) + \frac{p_1 r_1}{r_1+r_2}\left(1 - e^{-(r_1+r_2)}\right) \text{ if } t \geq 1 \text{ hour}$$

$$E = p_2 V_r^* \left(1 - e^{-r_2(1-t)} + e^{-r_2(1-t)}\left(1 - e^{-(r_1+r_2)t}\right)\frac{r_2}{r_1+r_2}\right)$$

$$+ p_1 e^{-r_2(1-t)}\left(1 - e^{-(r_1+r_2)t}\right)\frac{r_1}{r_1+r_2} \text{ if } t < 1 \text{ hour, and } V_r > 1$$

$$E = p_1\left(1 - e^{-r_1(1-t)} + e^{-r_1(1-t)}\left(1 - e^{-(r_1+r_2)t}\right)\frac{r_1}{r_1+r_2}\right)$$

$$+ p_2 V_r^* e^{-r_1(1-t)}\left(1 - e^{-(r_1+r_2)t}\right)\frac{r_2}{r_1+r_2} \text{ if } t < 1 \text{ hour, and } V_r < 1$$

Letting:

$$p_1 = p_2 = .10$$

$$r_1 = 2/\text{hour}$$

$$r_2 = 1/\text{hour}$$

Table 8 lists the results for several possible values of V_r^*.

While the scope of these investigations of value sensitivity is obviously limited, it appears that over-estimation of the value of targets of higher worth might impose less risk than under-estimation. Further examination in the area of sensitivity where several weapon types are employed simultaneously would be necessary before conclusive results could be stated.

V_r^*	If V_r^* is misestimated by a factor of z, the expected returns decrease by y.			
	z=10	z=5	z=1/2	z=1/10
	y	y	y	y
1	28%	22%	5%	8%
2	8%	6%	10%	37%
3	4%	3%	5%	53%
5	2%	1%	3%	67%
7	1%	1%	1%	66%
10	1%	<.5%	1%	40%

TABLE 8
PERCENTAGE DECREASE IN EXPECTED RETURNS
DUE TO MISESTIMATION OF V_r^*

4. CONCLUSIONS AND RECOMMENDATIONS

A method for determining an optimal initial loading for missions against targets of opportunity has been developed. As a by-product of this load determination, an optimal strategy for the sequential expenditure of weapons has been found. The gains in the expected value of targets killed, when the initial loading is optimal and when an optimal strategy is employed, have been shown to be significant when compared with initial loadings and firing doctrines derived by the application of simple intuition and judgement to kill probabilities, target values, and encounter rates.

The determination of an optimal loading and expenditure policy, however, are at present of such complexity as to require the use of a high speed computer in all but the most trivial cases. For those operational situations in which a computer will not be available, a hand-calculable approximation method for the determination of a best initial loading has also been developed, which, though sub-optimal, has been found to generate loads which yield high expected returns.

Either of the above methods should yield significantly better results than present policies which prescribe that aircraft be loaded for targets which are assumed known or that several aircraft each be given different loads in order to cover an unknown spectrum of targets.

The models presented in this paper assume that weapons can be loaded in any combination which satisfies rack and weight constraints, and that the pilot has complete freedom of choice as to the sequence in which weapons can be expended. For most attack aircraft these assumptions are not always true. Restrictions on unsymmetrical loadings

are one limiting factor. Others relate to the mechanics of multiple bomb racks. The models should be expanded to account for these aspects of the problem.

A vital assumption in the developments of both models was that inter-encounter times would be distributed exponentially. It is recommended that an analysis of operational data be made to ascertain the validity of this supposition.

It seems reasonable to assume that a list of all possible types of targets of opportunity, for a spectrum of scenarios, would be small enough - certainly not larger than 50 - that a thorough analysis, as suggested in Section 3, could be made of their relative values. Such an analysis would indicate general strategic values which could then be adjusted for specific tactical scenarios. Furthermore, additional investigations similar to that outlined in Section 3 should be undertaken to study the sensitivity of the optimal load and strategy calculations to these values.

The hand approximation method is rather crude in form. It is recommended that this model be examined further in an attempt to refine the calculations and to extend the model to cover a wider variety of cases in a more complete fashion.

The computer programs of Appendix II were written solely for the computations used in this thesis and should be improved upon before extensive investigations are made of the dynamic programming treatment of the problem. The four target-seven weapon example required two hours and fifteen minutes on a CDC-1604 computer for the determination of an optimal load. This is testimony to the inefficiency of the "BESTLD" FORTRAN program.

The problem of determining an optimal initial loading is similar to the classic "flyaway-kit" problem[1] of dynamic programming. In the strike aircraft loading problem the additional complication is present that the costs of shortages are not constant but depend on the initial loading. It would seem though that research into solution of the strike aircraft problem by this method should be fruitful. Very good approximations to an optimal initial loading should be obtainable very rapidly in this way.

Although the "POLICY" FORTRAN program computes the optimal strategy with relative ease, usually less than four minutes is required on a CDC-1604 computer, it is recommended that methods of approximation be developed for which a strategy can be calculated without the aid of a high speed computer.

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[1] Bellman, R. E. and Dreyfus, R. E., Applied Dynamic Programming, (Princeton, New Jersey: Princeton University Press, 1962), pp. 42-43.

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APPENDIX I

ENCOUNTER RATES

In a combat situation, many targets (especially "targets of opportunity") may be thought of as being distributed randomly throughout a given area, in that the pilot flying the mission does not know, beforehand, where or when a target will be encountered. This may be thought of as the classical situation in which it may be assumed that, in any small time interval dt, the probability that a target is encountered is proportional to dt and is independent of any previous encounters.

Over a period of time, photo or visual reconnaissance can give an average density of targets in a given area. From the attack aircraft speed and the average search sweep width (width of a path in which the pilot could be expected to see a target) the area swept per unit time can be obtained. By multiplying the average type i target density by the average sweep area per unit time, an estimate of the average encounter rate r_i is obtained for target type i.

The average encounter rate developed above can be used to generate the distribution of inter-encounter times.

Assumptions:

Probability of one encounter in dt = $r_i dt + h(dt)$ where $h(dt)$ approaches zero as dt approaches zero.

Probability of two or more encounters in dt = $h(dt)$

Probability of no encounters in dt = $1-r_i dt + h(dt)$ $p(t)dt$ = prob. ($t \leq$ first encounter time $\leq t + dt$) = $dP(t)$

$$P(t) = \int_0^t p(T)dt = \text{prob.(encounter time} \leq t)$$

$Q(t) = 1-P(t)$

Thus if t_e is the encounter time, and t is any arbitrary time in the mission,

$$Q(t+dt) = \text{prob.}((t_e > t)(\text{no encounter in } dt))$$

$$= Q(t)\ (1-r_i dt + h(dt))$$

Subtracting $Q(t)$ from both sides, dividing through by dt and taking the limit as dt approaches zero gives $dQ(t)/dt = -Q(T)r_i$

Rearranging gives $dQ(t)/Q(t) = -r_i dt$

Integrate both sides to get $\ln Q(t) = C-r_i t$

Thus $Q(t) = Ae^{-r_i t}$

But $Q(0) = 1$ since there can certainly be no encounters in zero search time. Therefore $A = 1$ and $Q(t) = e^{-r_i t}$ and consequently $P(t) = 1-e^{-r_i t}$ and $p(t) = r_i e^{-r_i t}$

Thus it is seen that the inter-encounter times are exponentially distributed. It also follows naturally that the probability distribution on the number of targets type i encountered during a mission of length T is Poisson,[1] i.e.,

$$\text{prob. } (N(T) = n) = (r_i T)^n\ e^{-r_i T}/n!$$

and the expected no. $\overline{N}(T) = r_i T$ (This is obtained from the observations).

[1]Parzen, E., Modern Probability Theory and Its Applications, (New York: John Wiley and Sons, 1960) p. 253.

APPENDIX II

BESTLD AND POLICY FORTRAN PROGRAMS

```
      PROGRAM BESTLD
      DIMENSION XLAM(10),P(8,8,10),V(10),X(512),F(512),N(8),W(8),J(9),
     1          ISUBS(512)
C     MAXIMUM OF 8 TARGET TYPES, 8 WEAPON TYPES, AND 10 WEAPON RACKS
      READ 551,L,LL,WM
  551 FORMAT (2I2,F10.0)
C    L IS THE NUMBER OF WEAPON RACKS, LL IS THE NUMBER OF WEAPON TYPES,
C     WM IS THE MAXIMUM PAYLOAD OF THE STRIKE AIRCRAFT
      READ 552,(W(I),I=1,8)
  552 FORMAT (8F6.0)
C     W(I) IS THE WEIGHT OF WEAPON TYPE I, IF THE NUMBER OF WEAPON TYPES
C     IS LESS THAN 8, SET W(LL+1),W(LL+2),....,W(8) EQUAL TO WM+1.
      READ 200, M, DELTA, T
  200 FORMAT (I5,2F5.0)
C     M IS THE NUMBER OF TARGET TYPES, DELTA IS DELTA T,
C     T IS THE MISSION TIME
      READ 202,(XLAM(I),V(I),I=1,M)
  202 FORMAT (2F5.0)
C     XLAM(I) IS THE ENCOUNTER RATE WITH TARGET TYPE I,
C     V(I) IS THE VALUE OF TARGET TYPE I
      DO 825 K=1,M
      DO 825 JN=1,LL
      READ 826, (P(I,JN,K),I=1,L)
```

```
  826 FORMAT (8F5.0)
C     P(I,JN,K) = THE PROBABILITY THAT I WEAPONS OF TYPE JN WILL KILL A
C     TARGET OF TYPE K.
  825 CONTINUE
      PRINT 553,(W(I),I=1,8)
  553 FORMAT(1H1,18X,1H1,6X,1H2,6X,1H3,6X,1H4,6X,1H5,6X,1H6,6X,1H7,6X,
     1          1H8,/15H WEAPON WEIGHTS,8F7.0////)
      PRINT555,L,WM
  555 FORMAT (19H NUMBER OF RACKS IS,I2,15H MAX PAYLOAD IS,F6.0,////)
      ZZ=0.0
      K5=0
      PRINT 304,M,T,DELTA
  304 FORMAT (1X,25HNUMBER OF TARGET TYPES IS,I2,/,
     1          16H MISSION TIME IS,F5.2,6H HOURS,/,11H DELTA T IS,F5.3//)
      PRINT 305
  305 FORMAT (12H TARGET TYPE,4X,14HENCOUNTER RATE,4X,5HVALUE)
      PRINT 306,(I,XLAM(I),V(I),I=1,M)
  306 FORMAT (I7,10X,F10.5,6X,F5.2)
      PRINT 837,(I,I=1,L)
  837 FORMAT (///,27X,18HKILL PROBABILITIES,/,6X,19HNUMBER OF WEAPONS =,
     1          4X,I1,7I6)
      DO 835 K=1,M
      DO 835 JN=1,LL
      PRINT 836,K,JN,(P(I,JN,K),I=1,L)
  836 FORMAT (7H TARGET,I3,3X,6HWEAPON,I2,4X,8F6.3)
  835 CONTINUE
      PRINT 733
  733 FORMAT (1H1)
      DO 500 I=2,8
  500 A=MIN1F(W(1),W(I))
      AMIN=WM-A+1.0
      DO 501 J1P = 1,9
```

```
      J(1)=J1P-1
      J2T=9-J(1)
      DO 501 J2P=1,J2T
      J(2)=J2P-1
      J3T=J2T-J(2)
      DO 501 J3P=1,J3T
      J(3)=J3P-1
      J4T=J3T-J(3)
      DO 501 J4P=1,J4T
      J(4)=J4P-1
      J5T=J4T-J(4)
      DO 501 J5P=1,J5T
      J(5)=J5P-1
      J6T=J5T-J(5)
      DO 501 J6P=1,J6T
      J(6)=J6P-1
      J7T=J6T-J(6)
      DO 501 J7P=1,J7T
      J(7)=J7P-1
      J8T=J7T-J(7)
      DO 501 J8P=1,J8T
      J(8)=J8P-1
      WLOAD=0.0
      NLOAD = 0
      DO 502 I=1,8
      NLOAD = NLOAD + J(I)
  502 WLOAD=WLOAD+J(I)*W(I)
      IF (NLOAD .LE. L) 823,501
  823 IF (WLOAD .GE. AMIN) 503,506
  506 IF (NLOAD .EQ. L) 504,501
  503 IF(WLOAD .LE. WM) 504,501
  504 N1P = J(1) + 1
```

```
      N2P = J(2) + 1
      N3P = J(3) + 1
      N4P = J(4) + 1
      N5P = J(5) + 1
      N6P = J(6) + 1
      N7P = J(7) + 1
      N8P = J(8) + 1
      I99 = 6
      IPROD = N1P * N2P * N3P * N4P * N5P * N6P * N7P * N8P
      DO 101 I=1,IPROD
  101 X(I) = 0.0
       JJ=0
      DO 1000 NN1 = 1,N1P
      I1 = NN1 - 1
      DO 1000 NN2 = 1,N2P
      I2 = NN2 - 1
      DO 1000 NN3 = 1,N3P
      I3 = NN3 -1
      DO 1000 NN4 = 1,N4P
      I4 = NN4-1
      DO 1000 NN5 = 1,N5P
      I5 = NN5 - 1
      DO 1000 NN6=1,N6P
      I6 = NN6 - 1
      DO 1000 NN7 =1,N7P
      I7 = NN7-1
      DO 1000 NN8 =1,N8P
      I8 = NN8-1
      JPROD =I1 * 10**7 +I2 * 10**6+ I3 * 10**5 + I4 * 10**4
     1+ I5 * 10**3 + I6 * 10**2  + I7 * 10 + I8
       JJ = JJ + 1
 1000 ISUBS(JJ) = JPROD
```

```
      LOAD = ISUBS(IPROD)
   58 SUMLAM = 0.0
      DO 100 I=1,M
  100 SUMLAM=SUMLAM+ XLAM(I)
      TEMP = 1.0 - DELTA* SUMLAM
      XNDIV =T/DELTA
      NDIV = XNDIV + 1
      DO 102 I1=2,NDIV
      TIME =(I1-1)*DELTA
C     THIS IS THE TIME REMAINING IN WHICH TO OPERATE
  721 DO 103 I2 = 2,IPROD
      ILOAD =ISUBS(I2)
      ITEM = ILOAD
      DO 625 IY =1,8
      N(IY) = ITEM/10**(8-IY)
  625 ITEM = ITEM - N(IY) * 10**(8-IY)
      TERM1= TEMP *X(I2)
      TERM2=0.0
      DO 104 I3=1,M
      Y=0.0
      DO 105 I4=1,8
      A=0.0
      NL = N(I4) + 1
      IF (NL-1) 105,105,107
  107 DO 106 I5=1,NL
      JI4 =I5-1
C     JI4 IS THE NUMBER OF WEAPONS OF TYPE I4 EXPENDED
      IF(JI4) 640,640,641
  640 FIRST = 0.0
      GO TO 642
  641 FIRST = P(JI4,I4,I3) * V(I3)
  642 JSUBS = ILOAD - JI4*10 **(8-I4)
```

```
      DO 108 I6 = 1,IPROD
      IF (JSUBS - ISUBS(I6)) 108,109,108
  109 I7 = I6
      GO TO 110
  108 CONTINUE
  110 SECOND=X(I7)
      Z=FIRST + SECOND
      IF (Z-A)106,111,111
  111 A = Z
  106 CONTINUE
      IF (A-Y) 105,112,112
  112 Y = A
  105 CONTINUE
  104 TERM2 = TERM2 + Y * XLAM(I3)*DELTA
      F(I2)=TERM1 + TERM2
  103 CONTINUE
      IF (I99 - I1) 129,129,128
  129 I99 = I99 + 5
      PRINT 320,(N(IY),IY=1,8),TIME,F(IPROD)
  320 FORMAT (1X,1HF,8I1,1H(,F5.3,3H) =,F10.5)
  128 DO 113 IT=2,IPROD
  113 X(IT) = F(IT)
  102 CONTINUE
      PRINT 888
  888 FORMAT (////)
      IF (F(IPROD) - ZZ) 501,501,141
  141 ZZ = F(IPROD)
      K5 = 10*LOAD + 10**10
  501 CONTINUE
      PRINT 666,K5,ZZ
  666 FORMAT(1H1,14H BEST LOAD IS ,I9,28H  WITH AN EXPECTED RETURN OF,
     1       F9.5)
      END
```

```
F00000120( .050) =    .34921
F00000120( .100) =    .56935
F00000120( .150) =    .76357
F00000120( .200) =    .94750
F00000120( .250) =   1.12276


F00000310( .050) =    .24291
F00000310( .100) =    .37144
F00000310( .150) =    .47282
F00000310( .200) =    .57092
F00000310( .250) =    .66664
```

BESTLD SAMPLE OUTPUT

TABLE 9

```
      PROGRAM POLICY
      DIMENSION XLAM(10),P(8,8,10),V(10),X(512),F(512),N(8),
     1     ISUBS(512),KFIRE(9,10),JFIRE(9),LFIRE(9)
      READ 200,M,N1,N2,N3,N4,N5,N6,N7,N8,DELTA,T
  200 FORMAT( 9I3,2F5.0)
C    M IS THE NUMBER OF TARGET TYPES
C    N1 IS THE NUMBER OF WEAPONS OF TYPE 1, ETC.
C    IF THERE ARE LESS THAN 8 WEAPON TYPES,SET N8 = 0,ETC.
C    DELTA IS DELTA T, T IS THE MISSION LENGTH.
      READ 202,(XLAM(I),V(I),I=1,M)
  202 FORMAT (2F5.0)
C    XLAM(I) IS THE ENCOUNTER RATE WITH TYPE I TARGETS,
C    V(I) IS THE VALUE OF TARGET TYPE I.
      READ 777,L,LL
  777 FORMAT (2I2)
C   L IS THE NUMBER OF WEAPON RACKS, LL IS THE NUMBER OF WEAPON TYPES
      DO 825 K=1,M
      DO 825 JN=1,LL
      READ 826, (P(I,JN,K),I=1,L)
  826 FORMAT (8F5.0)
C    P(I,JN,K) = THE PROBABILITY THAT I WEAPONS OF TYPE JN WILL KILL A
C    TARGET OF TYPE K.
  825 CONTINUE
      N1P=N1+1
      N2P=N2+1
      N3P=N3+1
      N4P=N4+1
      N5P=N5+1
      N6P=N6+1
      N7P=N7+1
      N8P=N8+1
      IPROD = N1P * N2P * N3P * N4P * N5P * N6P * N7P * N8P
```

```
      DO 101 I=1,IPROD
  101 X(I) = 0.0
      JJ=0
      DO 1000 NN1 = 1,N1P
      I1 = NN1 - 1
      DO 1000 NN2 = 1,N2P
      I2 = NN2 - 1
      DO 1000 NN3 = 1,N3P
      I3 = NN3 -1
      DO 1000 NN4 = 1,N4P
      I4 = NN4-1
      DO 1000 NN5 = 1,N5P
      I5 = NN5 - 1
      DO 1000 NN6=1,N6P
      I6 = NN6 - 1
      DO 1000 NN7 =1,N7P
      I7 = NN7-1
      DO 1000 NN8 =1,N8P
      I8 = NN8-1
      JPROD =I1 * 10**7 +I2 * 10**6+ I3 * 10**5 + I4 * 10**4
     1+ I5 * 10**3 + I6 * 10**2  + I7 * 10 + I8
      JJ = JJ + 1
 1000 ISUBS(JJ) = JPROD
      LOAD = ISUBS(IPROD)
      PRINT  304, M,LOAD,T,DELTA
  304 FORMAT(1H1,25HNUMBER OF TARGET TYPES IS,I2,/,20H INITIAL LOADING
     1IS,I9,/,16H MISSION TIME IS,F5.2,6H HOURS,/,11H DELTA T IS,F5.3//)
      PRINT 305
  305 FORMAT (12H TARGET TYPE,4X,14HENCOUNTER RATE,4X,5HVALUE)
      PRINT 306,(I,XLAM(I),V(I),I=1,M)
  306 FORMAT (I7,10X,F10.5,6X,F5.2)
      PRINT 837
```

```
837 FORMAT (1H1,40X,17HNUMBER OF WEAPONS)
    DO 835 K=1,M
    DO 835 JN=1,LL
    PRINT 836,K,JN,(P(I,JN,K),I=1,L)
836 FORMAT (12H TARGET TYPE,I3,3X,12HWEAPONS TYPE,I2,6X,3F8.4)
835 CONTINUE
    SUMLAM=0.0
    DO 100 I=1,M
100 SUMLAM=SUMLAM+ XLAM(I)
    TEMP = 1.0 - DELTA* SUMLAM
    XNDIV =T/DELTA
    NDIV = XNDIV + 1.0
    DO 102 I1=2,NDIV
    TIME =(I1-1)*DELTA
C     THIS IS THE TIME REMAINING IN WHICH TO OPERATE
    PRINT 301,TIME
301 FORMAT (//,17H TIME REMAINING =,F6.3)
    DO 103 I2=2,IPROD
    ILOAD =ISUBS(I2)
    ITEM = ILOAD
    DO 625 I=1,8
    N(I) = ITEM/10**(8-I)
625 ITEM = ITEM - N(I)*10**(8-I)
    TERM1= TEMP *X(I2)
    TERM2=0.0
    DO 104 I3=1,M
    Y=0.0
    DO 105 I4=1,8
    KFIRE(I4,I3)=0
C     KFIRE(I,K) INDICATES THE NUMBER OF WEAPONS OF TYPE I THAT SHOULD
C     BE EXPENDED IF A TARGET TYPE K IS ENCOUNTERED
    A=0.0
```

```
      NL = N(I4) + 1
      IF (NL-1) 105,105,107
  107 DO 106 I5=1,NL
      JI4 =I5-1
C      JI4 IS THE NUMBER OF WEAPONS OF TYPE I4 EXPENDED
      IF(JI4) 640,640,641
  640 FIRST = 0.0
      GO TO 642
  641 FIRST = P(JI4,I4,I3) * V(I3)
  642 JSUBS = ILOAD - JI4*10 **(8-I4)
      DO 108 I6 = 1,IPROD
      IF (JSUBS - ISUBS(I6)) 108,109,108
  109 I7 = I6
      GO TO 110
  108 CONTINUE
  110 SECOND=X(I7)
      Z=FIRST + SECOND
      IF (Z-A)106,111,111
  111 A = Z
      KFIRE (I4,I3) = JI4
  106 CONTINUE
      IF (A-Y) 105,112,112
  112 Y = A
      JFIRE(I3) = KFIRE(I4,I3)
      LFIRE(I3)=I4
  105 CONTINUE
  104 TERM2 = TERM2 + Y * XLAM(I3)*DELTA
      F(I2)=TERM1 + TERM2
      PRINT 302,(N(I),I=1,8)
  302 FORMAT (15H SUBLOADING IS ,8I1)
      PRINT 303, (I,JFIRE(I),LFIRE(I),I=1,M)
  303 FORMAT (15H IF TARGET TYPE,I2,20H IS ENCOUNTERED FIRE,I2,
```

```
    1       8H OF TYPE,I2)
103 CONTINUE
    PRINT 320,(N(IY),IY=1,8),TIME,F(IPROD)
320 FORMAT (//,1X,1HF,8I1,1H(,F5.3,3H) =,F10.5,//////)
    DO 113 I=2,IPROD
113 X(I)=F(I)
102 CONTINUE
    END
```

```
TIME REMAINING =    .212
SUBLOADING IS 00000010
IF TARGET TYPE 1 IS ENCOUNTERED FIRE 0 OF TYPE 7
IF TARGET TYPE 2 IS ENCOUNTERED FIRE 1 OF TYPE 7
IF TARGET TYPE 3 IS ENCOUNTERED FIRE 1 OF TYPE 7
IF TARGET TYPE 4 IS ENCOUNTERED FIRE 0 OF TYPE 7
SUBLOADING IS 00000020
IF TARGET TYPE 1 IS ENCOUNTERED FIRE 0 OF TYPE 7
IF TARGET TYPE 2 IS ENCOUNTERED FIRE 2 OF TYPE 7
IF TARGET TYPE 3 IS ENCOUNTERED FIRE 2 OF TYPE 7
IF TARGET TYPE 4 IS ENCOUNTERED FIRE 0 OF TYPE 7
```

POLICY SAMPLE OUTPUT

TABLE 10

APPENDIX III

TABULATED VALUES OF f_{ijs}

BJS	RT = .05	.10	.15	.20	.25
1	.04877	.09516	.13929	.18127	.22120
2	.04998	.09984	.14948	.19879	.24770
3	.05000	.10000	.14998	.19994	.24986
4	.05000	.10000	.15000	.20000	.24999
5	.05000	.10000	.15000	.20000	.25000
6	.05000	.10000	.15000	.20000	.25000
7	.05000	.10000	.15000	.20000	.25000
8	.05000	.10000	.15000	.20000	.25000
9	.05000	.10000	.15000	.20000	.25000
10	.05000	.10000	.15000	.20000	.25000
11	.05000	.10000	.15000	.20000	.25000
12	.05000	.10000	.15000	.20000	.25000
13	.05000	.10000	.15000	.20000	.25000
14	.05000	.10000	.15000	.20000	.25000
15	.05000	.10000	.15000	.20000	.25000
16	.05000	.10000	.15000	.20000	.25000
17	.05000	.10000	.15000	.20000	.25000
18	.05000	.10000	.15000	.20000	.25000
19	.05000	.10000	.15000	.20000	.25000
20	.05000	.10000	.15000	.20000	.25000

BJS	RT = .30	.35	.40	.45	.50
1	.25918	.29531	.32968	.36237	.39347
2	.29612	.34398	.39123	.43781	.48367
3	.29972	.34949	.39916	.44869	.49806
4	.29998	.34997	.39993	.44989	.49981
5	.30000	.35000	.40000	.44999	.49998
6	.30000	.35000	.40000	.45000	.50000
7	.30000	.35000	.40000	.45000	.50000
8	.30000	.35000	.40000	.45000	.50000
9	.30000	.35000	.40000	.45000	.50000
10	.30000	.35000	.40000	.45000	.50000
11	.30000	.35000	.40000	.45000	.50000
12	.30000	.35000	.40000	.45000	.50000
13	.30000	.35000	.40000	.45000	.50000
14	.30000	.35000	.40000	.45000	.50000
15	.30000	.35000	.40000	.45000	.50000
16	.30000	.35000	.40000	.45000	.50000
17	.30000	.35000	.40000	.45000	.50000
18	.30000	.35000	.40000	.45000	.50000
19	.30000	.35000	.40000	.45000	.50000
20	.30000	.35000	.40000	.45000	.50000

TABLE 11
VALUES OF f_{ijs}

BJS	RT = .55	.60	.65	.70	.75
1	.42305	.45119	.47795	.50341	.52763
2	.52878	.57309	.61658	.65922	.70099
3	.54724	.59621	.64492	.69336	.74150
4	.54971	.59956	.64937	.69911	.74879
5	.54997	.59996	.64993	.69990	.74985
6	.55000	.60000	.64999	.69999	.74998
7	.55000	.60000	.65000	.70000	.75000
8	.55000	.60000	.65000	.70000	.75000
9	.55000	.60000	.65000	.70000	.75000
10	.55000	.60000	.65000	.70000	.75000
11	.55000	.60000	.65000	.70000	.75000
12	.55000	.60000	.65000	.70000	.75000
13	.55000	.60000	.65000	.70000	.75000
14	.55000	.60000	.65000	.70000	.75000
15	.55000	.60000	.65000	.70000	.75000
16	.55000	.60000	.65000	.70000	.75000
17	.55000	.60000	.65000	.70000	.75000
18	.55000	.60000	.65000	.70000	.75000
19	.55000	.60000	.65000	.70000	.75000
20	.55000	.60000	.65000	.70000	.75000

BJS	RT = .80	.85	.90	.95
1	.55067	.57259	.59343	.61326
2	.74188	.78187	.82095	.85911
3	.78930	.83675	.88380	.93045
4	.79838	.84788	.89726	.94653
5	.79979	.84971	.89961	.94948
6	.79998	.84997	.89995	.94993
7	.80000	.85000	.89999	.94999
8	.80000	.85000	.90000	.95000
9	.80000	.85000	.90000	.95000
10	.80000	.85000	.90000	.95000
11	.80000	.85000	.90000	.95000
12	.80000	.85000	.90000	.95000
13	.80000	.85000	.90000	.95000
14	.80000	.85000	.90000	.95000
15	.80000	.85000	.90000	.95000
16	.80000	.85000	.90000	.95000
17	.80000	.85000	.90000	.95000
18	.80000	.85000	.90000	.95000
19	.80000	.85000	.90000	.95000
20	.80000	.85000	.90000	.95000

BJS	RT = 1	2	3	4	5
1	.63212	.86466	.95021	.98168	.99326
2	.89636	1.45866	1.75106	1.89011	1.95283
3	.97666	1.78198	2.32787	2.65200	2.82818
4	.99565	1.92486	2.68064	3.21853	3.56316
5	.99931	1.97751	2.86538	3.58970	4.12266
6	.99991	1.99408	2.94930	3.80457	4.50670
7	.99999	1.99861	2.98281	3.91524	4.74452
8	1.00000	1.99971	2.99471	3.96637	4.87789
9	1.00000	1.99994	2.99851	3.98774	4.94598
10	1.00000	1.99999	2.99962	3.99587	4.97781
11	1.00000	2.00000	2.99991	3.99871	4.99151
12	1.00000	2.00000	2.99998	3.99962	4.99696
13	1.00000	2.00000	3.00000	3.99990	4.99898
14	1.00000	2.00000	3.00000	3.99997	4.99968
15	1.00000	2.00000	3.00000	3.99999	4.99990
16	1.00000	2.00000	3.00000	4.00000	4.99997
17	1.00000	2.00000	3.00000	4.00000	4.99999
18	1.00000	2.00000	3.00000	4.00000	5.00000
19	1.00000	2.00000	3.00000	4.00000	5.00000
20	1.00000	2.00000	3.00000	4.00000	5.00000

BJS	RT = 6	7	8	9	10
1	.99752	.99909	.99966	.99988	.99995
2	1.98017	1.99179	1.99665	1.99864	1.99946
3	2.91820	2.96216	2.98289	2.99241	2.99669
4	3.76700	3.88039	3.94051	3.97118	3.98635
5	4.48194	4.70740	4.84088	4.91622	4.95710
6	5.03626	5.40669	5.64964	5.80053	5.89001
7	5.42996	5.95698	6.33627	6.59375	6.75987
8	5.68598	6.35827	6.88331	7.26985	7.53965
9	5.83874	6.62918	7.29076	7.81420	8.20663
10	5.92267	6.79868	7.57414	8.22679	8.74890
11	5.96529	6.89720	7.75825	8.52080	9.16586
12	5.98538	6.95055	7.87017	8.71779	9.46908
13	5.99421	6.97755	7.93397	8.84202	9.67753
14	5.99783	6.99036	7.96815	8.91587	9.81306
15	5.99923	6.99608	7.98541	8.95734	9.89652
16	5.99974	6.99848	7.99364	8.97937	9.94526
17	5.99992	6.99944	7.99736	8.99048	9.97230
18	5.99998	6.99980	7.99895	8.99580	9.98658
19	5.99999	6.99993	7.99960	8.99823	9.99377
20	6.00000	6.99998	7.99986	8.99928	9.99722

BJS	RT = 11	12	13	14	15
1	.99998	.99999	1.00000	1.00000	1.00000
2	1.99978	1.99991	1.99997	1.99999	1.99999
3	2.99857	2.99939	2.99974	2.99989	2.99996
4	3.99366	3.99710	3.99869	3.99942	3.99974
5	4.97855	4.98950	4.99495	4.99761	4.99869
6	5.94103	5.96916	5.98422	5.99208	5.99610
7	6.86242	6.92334	6.95833	6.97785	6.98846
8	7.71923	7.83383	7.90430	7.94623	7.97046
9	8.48724	8.67880	8.80455	8.88418	8.93301
10	9.14673	9.43641	9.63873	9.77478	9.86316
11	9.68684	10.08918	10.38705	10.59910	10.74470
12	10.10757	10.62759	11.03389	11.33906	11.55994
13	10.41888	11.05162	11.57078	11.98060	12.29233
14	10.63759	11.37008	11.99774	12.51615	12.92912
15	10.78354	11.59806	12.32261	12.94572	13.46346
16	10.87615	11.75364	12.55900	13.27636	13.89537
17	10.93207	11.85493	12.72351	13.52044	14.23125
18	10.96426	11.91790	12.83304	13.69324	14.48239
19	10.98195	11.95532	12.90287	13.81059	14.66292
20	10.99124	11.97660	12.94554	13.88710	14.78770

BJS	RT = 16	17	18	19	20
1	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.00000	2.00000	2.00000	2.00000	2.00000
3	2.99998	2.99999	3.00000	3.00000	3.00000
4	3.99989	3.99995	3.99998	3.99999	4.00000
5	4.99949	4.99977	4.99990	4.99995	4.99998
6	5.99810	5.99909	5.99957	5.99980	5.99991
7	6.99410	6.99703	6.99853	6.99928	6.99965
8	7.98410	7.99160	7.99563	7.99777	7.99887
9	8.96211	8.97900	8.98858	8.99389	8.99679
10	9.91881	9.95288	9.97320	9.98504	9.99179
11	10.84142	10.90375	10.94283	10.96672	10.98098
12	11.71442	11.81908	11.88794	11.93204	11.95959
13	12.52130	12.68406	12.79627	12.87148	12.92058
14	13.24679	13.48319	13.65368	13.77308	13.85445
15	13.87926	14.20235	14.44560	14.62333	14.74959
16	14.41252	14.83090	15.15895	15.40854	15.59308
17	14.84656	15.36316	15.78390	16.11651	16.37200
18	15.18721	15.79914	16.31525	16.73814	17.07497
19	15.44486	16.14418	16.75300	17.26866	17.69355
20	15.63262	16.40786	17.10209	17.70805	18.22329

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A pilot engaged in a strike mission against targets of opportunity is continually confronted with the decision as to how many and which of his available weapons should be expended against a variety of targets which are encountered. Associated with this decision problem is the problem of determining an optimal load for such a mission subject to the constraints of available payload and number of weapons stations on the aircraft. Certain assumptions are made concerning the distribution of targets within the target area which lead to a dynamic programming formulation of the decision problem. This yields a system of ordinary differential equations which are solvable recursively. In addition to the dynamic programming model, a sub-optimum determination of a "best load" is discussed. Although not as complete or precise as the dynamic programming method, this formulation [illegible] adaptable to squadron level decision making.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
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Weapons						
Selection						
Allocation						
Firing						
Strategy						
Targets						
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